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LETTER TO THE EDITOR

Surface Hall effect in magneto-electric media

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Abstract. The nature of the bulk energy per unit volume in magneto-electric crystals implies an unusual form of Hall effect confined to the boundary surface crystal faces.

The most interesting features of the Hall effect are easily deduced by purely thermodynamic reasoning (Widom 1982). This is especially true for magneto-electric phenomena (Landau *et al* 1984) which have been of very long standing interest (O'Dell 1970). The purpose of this letter is to note that the bulk adiabatic magneto-electric tensor completely determines the boundary surface Hall impedance in a magneto-electric crystal. This result is worthy of note for laboratory determinations of this important tensor.

Recall that the adiabatic magneto-electric tensor is defined by

$$-\alpha_{ij} = (\partial^2/\partial E_i \partial B_j)\, U(\boldsymbol{E}, \boldsymbol{B}, S) \tag{1}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ represent the electromagnetic fields, S represents the entropy per unit volume, and U represents the energy per unit volume of the magneto-electric crystal.

Alternatively, with $\boldsymbol{P}$ as the electric dipole moment per unit volume, and $\boldsymbol{M}$ as the magnetic moment per unit volume,

$$\alpha_{ij} = (\partial M_j/\partial E_i)_{S,\boldsymbol{B}} = (\partial P_i/\partial B_j)_{S,\boldsymbol{E}}. \tag{2}$$

Now, let $\boldsymbol{n}$ be a unit vector normal to a crystal face on the boundary surface of a magneto-electric crystal. By virtue of the dipole moment per unit volume in the bulk crystal, the boundary face will develop a polarisation charge density per unit area given by

$$\sigma = -\boldsymbol{n} \cdot \boldsymbol{P}. \tag{3}$$

By virtue of a previously proven Nyquist theorem (Widom 1984), the boundary surface Hall impedance R_{H} is determined by

$$-R_{\mathrm{H}}^{-1} = c\boldsymbol{n} \cdot (\partial\sigma/\partial\boldsymbol{B})_{S,\boldsymbol{E}}. \tag{4}$$

Thus, from equations (3) and (4), one obtains the extremely simple (yet totally rigorous) result that the Hall impedance on the boundary surface face of a magneto-electric crystal is determined by the bulk adiabatic tensor $\boldsymbol{\alpha}$ in equation (2), i.e.

$$R_{\mathrm{H}}^{-1} = c\boldsymbol{n} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{n}. \tag{5}$$

Equation (5) is the central result of this work.

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In terms of the vacuum impedance

$$R_{\text{vac}} = (4\pi/c) \tag{6}$$

equation (5) reads

$$(R_{\text{vac}}/R_{\text{H}}) = 4\pi(\boldsymbol{n}\cdot\boldsymbol{\alpha}\cdot\boldsymbol{n}). \tag{7}$$

Thus, a surface Hall impedance measurement (usually expected to be large on a scale of $R_{\text{vac}} \simeq 377\ \Omega$) determines the adiabatic magneto-electric tensor α_{ij} component with both indices in the normal direction $\boldsymbol{n}$ as in equation (7).

Finally, it is quite interesting to note physical similarities between magneto-electric phenomena and spin-wave vacuum polarisation (Widom *et al* 1985).

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